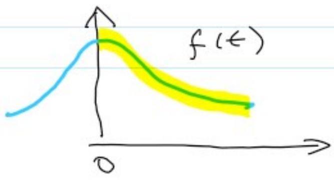
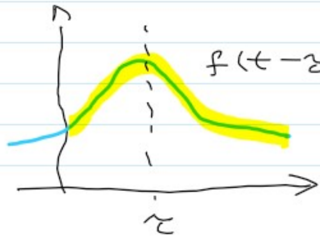
Konzultace 5. a 6. přednáška

úterý 7. dubna 2020 13:08

$$\mathcal{L}\{\mathbb{1}(t-\tau)\, f(t-\tau)\} = e^{-p\tau} F(p)$$

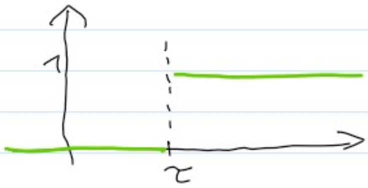


$$F(p) = \int_0^\infty f(t) \cdot e^{-pt}\, dt =$$



$$\mathcal{L}\{f(t-\tau)\}$$

$$\mathcal{L}\{t^2+2t+1\}$$

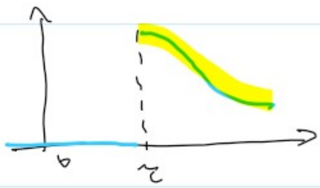


$$\mathcal{L}\{\underline{\mathbb{1}(t-2)}\,(\underbrace{t^2+2t+1})\}$$

$(t+2)-2$

$$\begin{cases} f(t-2) = t^2+2t+1 \\ f(t) = (t+2)^2 + 2\cdot(t+2) + 1 \\ \quad = \dots \end{cases}$$

$$e^{-2p} \cdot \mathcal{L}\{f(t)\}$$



$\delta(t)$

$$y''(t) + 3\cdot y'(t) + 2y(t) = u(t) \quad ; \; y(0) = 1, \; y'(0) = -1$$

$$\mathcal{L}\{y(t)\} \to Y(p)$$

$$\mathcal{L}\{y'(t)\} \to p\cdot Y(p) - y(0)$$

$$\mathcal{L}\{y''(t)\} \to p^2 Y(p) - p\cdot y(0) - y'(0)$$

$$p^2\cdot Y(p) - p\cdot y(0) - y'(0) + 3[p\cdot Y(p) - y(0)] + 2Y(p) = U(p)$$

$$p^2 Y(p) - p + 1 + 3pY(p) - 3 + 2Y(p) = U(p)$$

$$Y(p)[p^2+3p+2] = U(p) + p + 2$$

$$Y(p) = \frac{U(p)+p+2}{p^2+3p+2} \qquad \left\{ \frac{p+3}{p^2+3p+2} \right.$$

$$y''(t) + 3y'(t) + 2y(t) = 5\cdot u(t)$$

$$u(t) = \mathbb{1}(t) \qquad \mathcal{L}\{\mathbb{1}(t)\} = \frac{1}{p}$$

$$y(0) = -1$$

$$y'(0) = 2$$

$$y'(0) = 2$$

$$p^2 \cdot Y(p) - p \cdot y(0) - y'(0) + 3\big(p \cdot Y(p) - y(0)\big) + 2 \cdot Y(p) = 5\,\frac{1}{p}$$

$$p^2 \cdot Y(p) + p - 2 + 3\,p\,Y(p) + 3 + 2 \cdot Y(p) = \frac{5}{p}$$

$$Y(p)\,(p^2 + 3p + 2) + p + 1 = \frac{5}{p}$$

$$Y(p)\,(p^2 + 3p + 2) = \frac{5}{p} - p - 1$$

$$Y(p)\,(p^2 + 3p + 2) = \frac{5 - p^2 - p}{p}$$

$$Y(p) = \frac{-p^2 - p + 5}{p\,(p^2 + 3p + 2)} = \frac{-p^2 - p + 5}{p\,(p+1)(p+2)}$$

$$\frac{-p^2 - p + 5}{p(p+1)(p+2)} = \frac{A}{p} + \frac{B}{p+1} + \frac{C}{p+2}$$

$$A: \lim_{p \to 0} \frac{-p^2 - p + 5}{(p+1)(p+2)} = \frac{5}{2}$$

$$B: \lim_{p \to -1} \frac{-p^2 - p + 5}{p(p+2)} = \frac{-1 + 1 + 5}{-1(1)} = \frac{5}{-1} = -5$$

$$C: \lim_{p \to -2} \frac{-p^2 - p + 5}{p(p+1)} = \frac{-4 + 2 + 5}{-2(-1)} = \frac{3}{2}$$

$$Y(p) = \frac{5}{2} \cdot \frac{1}{p} - 5\,\frac{1}{p+1} + \frac{3}{2}\,\frac{1}{p+2}$$

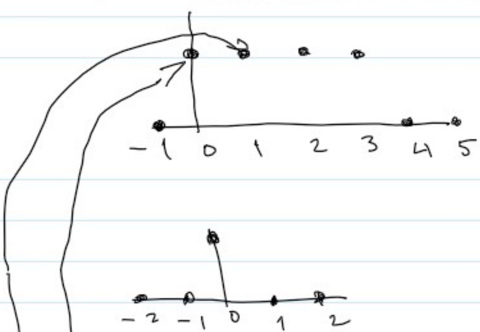
$$\left\{ \begin{array}{l} \mathcal{L}\{\mathbb{1}(t)\} \to \frac{1}{p} \\ \mathcal{L}\{e^{-\alpha t}\} \to \frac{1}{p+\alpha} \end{array} \right.$$

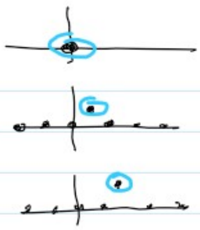
$$\mathcal{L}^{-1}\{Y(p)\} \to y(t) = \frac{5}{2} \cdot \mathbb{1}(t) - 5\,e^{-t} + \frac{3}{2} \cdot e^{-2t}$$

$$\mathbb{1}[n]$$

$$\left\{ \underline{\sin(t)} \cdot \mathbb{1}[n-1] \right.$$

$$\underline{y[n] = x[n] * h[n]}$$





$$\Sigma$$

$$x[n] \cdot \delta[n] \quad \longleftrightarrow \quad \left| \begin{array}{l} x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] \\ h[n] = \delta[n] + \delta[n-1] + \delta[n-2] \end{array} \right.$$

$$x[n] \cdot \delta[n-1]$$

$$\vdots$$

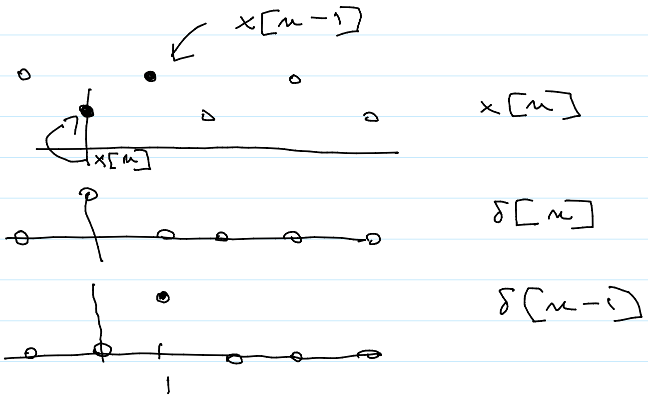
$$y[n] = x[n] * h[n] = x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2]\} =$$

$$= x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2] =$$

$$= x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2] =$$
$$= x[n] + x[n-1] + x[n-2]$$

$$y[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] +$$
$$\delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$$
$$+ \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

$$y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + \delta[n-5]$$



$$y[n] = \{1; 2; 3; 3; 2; 1\}$$